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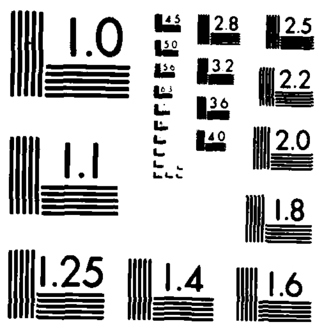

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CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
University of North Carolina
Chapel Hill, North Carolina

ON STOCHASTIC INTEGRAL REPRESENTATION OF STABLE

PROCESSES WITH SAMPLE PATHS IN BANACH SPACES

by

Jan Rosinski

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$$X(t) = \int_S h(t,s)\, dM(s), \quad t \in T,$$

are studied in terms of the kernel h. The existence of an appropriate modification of the kernel h enables one to use results from stable measures on Banach spaces in studying of X. Bounds for the moments of the norm of sample paths of X are obtained. This yields definite bounds for the moments of a double α-stable integral. Also necessary and sufficient conditions for the absolute continuity of sample paths of X are given. Along with the above stochastic integral representation of stable processes, the representation of stable random vectors due to LePage, Woodrooffe and Zinn is extensively used and the relationship between these two representations is discussed.

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1. Introduction.

The Central Limit Theorem and the stability property provide the basic reasons for regarding stable processes as a natural generalization of Gaussian ones. As an analog to the well-known spectral representation of stationary Gaussian processes, every *symmetric α-stable* (SαS) stochastic process with parameter set T has a version of the form

$$(1.1) \qquad X(t) = \int_S h(t,s)dM(s), \qquad t \in T$$

(cf. [1], [12], [29], [30], [11] and the discussion of the history of (1.1) in [9]) and in the stationary case one can choose $t \to h(t,\cdot)$ as an orbit of a group of isometries on L^α (see [9]). Here M is an independently scattered SαS random measure on an abstract measurable space (S,A).

There are two special cases of (1.1) that have been extensively studied: *harmonizable processes* given by

$$(1.2) \qquad X(t) = \int_{-\infty}^{\infty} e^{its}dM(s), \qquad t \in \mathbb{R}$$

(with appropriate modifications if t runs over a group) (cf. [10], [19], [6], [17], [2], [23] and [32]) and *moving averages*

$$(1.3) \qquad X(t) = \int_{-\infty}^{\infty} g(t-s)dM(s), \qquad t \in \mathbb{R}$$

(cf. [6], [25], [4] and [2]).

In this paper we study general SαS processes given by (1.1). They are determined by two quantities: the kernel h and the control measure m of M. In contrast with the approach taken in [9] and [2], which relies on the properties of the mapping $T \ni t \to h(t,.) \in L^\alpha$, we relate path properties of X with properties of the mapping $S \ni s \to h(.,s) \in \mathbb{R}^T(\mathbb{C}^T)$ which plays the crucial role here. More specifically, we are concerned with processes having sample paths in a separable Banach space $V(T)$ of functions defined on T. We show that the kernel h in (1.1) admits a modification with all sections $h(.,s)$ in $V(T)$ (Section 5). Therefore we may always replace (1.1) by

$$X = \int_S h(.,s)dM(s), \tag{1.4}$$

where on the right-hand side we have a stochastic integral of the $V(T)$-valued function $s \to h(.,s)$. Such stochastic integrals of Banach space valued functions have been investigated in [26] for infinitely divisible random measures. In the present stable case the construction can be simplified and this is done at the beginning of Section 3. Then we establish the relationship between the stochastic integral representation of stable random vectors in Banach spaces and the series representation due to LePage, Woodrooffe and Zinn [13].

In Section 4 we use some ideas of Marcus and Pisier [17] and an adaptation of Hoffmann-Jørgensen's inequality due to Giné and Zinn [8] to obtain bounds for moments of the norm of $X(.)$. We also introduce a complete norm on the space of all vector valued functions f for which $\int f dM$ exists, similar to Pisier's norm for CLT [21]. Theorem 4.5 establishes the role of simple functions in the series representation of stable random vectors.

In Section 6 we apply the results of Sections 5 and 4 to characterize the absolute continuity of sample paths of $S\alpha S$ processes. Earlier the absolute continuity of sample paths has been investigated by Cambanis and Miller [3] in terms the so-called covariation function, only for the case $\alpha > 1$.

Continuing the above approach, we obtain in Section 7 definitive bounds for moments of a double α-stable stochastic integral. We also give a short and new proof of a Fubini-type result which allows the interchanging of stochastic and usual integration (cf. [4] Theorem 4.6 and [20] Lemma 4.4).

2. Preliminaries and notation.

A systematic treatment of stable measures on Banach spaces one can find in Linde [14] and we shall refer to this book for basic definitions and facts. The characteristic functional of an SαS p.m. μ on a separable Banach space B can be written in the form

$$\hat{\mu}(x') = \exp(-\int_{\partial U} |\langle x,x'\rangle|^{\alpha} d\sigma(x)), \tag{2.1}$$

$x' \in B'$, where ∂U is the unit sphere in B and σ is a finite symmetric measure on ∂U. σ is uniquely determined by μ and is called the *spectral measure of* μ (cf. Theorem 6.4.4 [14]). Further, for every $p \in (0,\alpha)$ $\int_B ||x||^p d\mu(x) < \infty$ and for every $p,q \in (0,\alpha)$

$$\left(\int_B ||x||^p d\mu(x)\right)^{1/p} \underset{C}{\sim} \left(\int_B ||x||^q d\mu(x)\right)^{1/q} \tag{2.2}$$

where $C = C(\alpha,p,q)$ (we shall write $L \underset{C}{\sim} R$ if $C^{-1}R \le L \le CR$ and $C = C(a,b,\dots)$ means that a positive constant C depends only on parameters $a,b,\dots$). If $\hat{\mu}$ is given by

$$\hat{\mu}(x') = \exp\left(-\int_B |\langle x,x'\rangle|^{\alpha} d\sigma_0(x)\right),$$

$x' \in B'$, where σ_0 is a (non-necessary symmetric) Borel measure on B, which is σ-finite on $B\setminus\{0\}$, then $\int_B ||x||^{\alpha} d\sigma_0(x) < \infty$ and for every $p \in (0,\alpha)$

$$\left(\int_B ||x||^{\alpha} d\sigma_0(x)\right)^{1/\alpha} \le C\left(\int_B ||x||^p d\mu(x)\right)^{1/p}, \tag{2.3}$$

where $C = C(\alpha,p)$ (cf. Proposition 6.4.5 and Corollary 7.3.5 in [14]).

Let A be a δ-ring of subsets of a non-empty set S (i.e. a ring that is closed under countable intersections). A stochastic process $\{M(A): A \in \mathcal{A}\}$ is said to be an $S\alpha S$ *random measure* if

(i) for every sequence $\{A_n\} \subset \mathcal{A}$ of pairwise disjoint sets with $\cup A_n \in \mathcal{A}$ the series $\Sigma\, M(A_n)$ converges in probability to $M(\cup A_n)$;

(ii) $M(A_1), M(A_2), \ldots$ are independent, provided $A_n \in \mathcal{A}$ are pairwise disjoint;

(iii) $M(A)$ has an $S\alpha S$ distribution for every $A \in \mathcal{A}$.

It follows that the characteristic function of $M(A)$ can be written in the form

$$E \exp (it\, M(A)) = \exp (-m(A)|t|^{\alpha}), \quad t \in \mathbf{R}, \quad A \in \mathcal{A},$$

where m is a non-negative measure on $\mathcal{A}$. m is called the *control measure of* M.

The existence of an $S\alpha S$ random measure M with a given control measure m follows by Kolmogorov's Consistency Theorem. In particular, if $X(s)$, $s \geq 0$ is an independent stationary increment process such that $E \exp (itX(s)) = \exp(-s|t|^{\alpha})$, then M defined on intervals by $M((a,b]) = X(b) - X(a)$ extends to an $S\alpha S$ random measure on the δ-ring of all Borel bounded subsets of $[0,\infty)$ with the Lebesgue measure as the control measure. (see Prekopa [22]).

Throughout this paper we shall assume that $(S,\mathcal{A})$ satisfies the following condition: there exists a sequence $\{A_n\} \subset \mathcal{A}$ such that $\cup A_n = S$. Then every countably additive finite measure on $\mathcal{A}$ extends uniquely to a σ-finite measure on $\sigma(\mathcal{A})$ and as a consequence every control measure of an $S\alpha S$ random measure M is the restriction to $\mathcal{A}$ of a σ-finite measure m defined on $\sigma(\mathcal{A})$. To avoid trivialities we shall always assume that m is not zero.

For every real function $f \in L^{\alpha}(S,\sigma(\mathcal{A}),m)$ the stochastic integral $\int_S f\,dM$ is defined as the limit in L^p of integrals of simple functions and satisfies the equality

$$(E\ |\int_S fdM|^p)^{1/p} = C\ (\int_S |f|^\alpha dM)^{1/\alpha}\ ,$$

where $C = C(\alpha,p)$, $p < \alpha$ (cf. [1], and [29]). If B is a finite dimensional Banach space, then for $f \in L_B^\alpha(S,\sigma(A),m)$

$$(2.4) \qquad (E\ ||\int_S fdM||^p)^{1/p} \underset{C}{\sim} (\int_S ||f||^\alpha dm)^{1/\alpha}\ ,$$

where C depends on α and p and additionally on dim B and $||\cdot||$. Banach spaces in which (2.4) holds for all simple functions with a universal constant C (and thus $\int_S fdM$ can be defined for all $f \in L_B^\alpha$) have been characterized by Marcus and Woyczynski [18]. This is the class of spaces of stable type α, including in particular Hilbert spaces and L^p-spaces for $p > \alpha$ (see [14] for further references).

3. Stochastic integral and series representation of SαS random vectors in Banach spaces.

Let M be an SαS random measure on (S,A) with the control measure m. Let L_B^{simple} be the space of all simple measurable functions f: $(S,\sigma(A)) \to (B, \text{Borel }(B))$ such that $\{s: f(s) \neq 0\} \in A$. As usual functions equal m-a.e. are indistinguishable. For every $f \in L_B^{simple}$, $f = \Sigma x_j I_{A_j}$, $x_j \neq 0$, $A_j \in A$ we set

$$\int_S fdM = \Sigma x_j M(A_j)$$

and

$$\lambda_{\alpha,p}(f) = (E\, ||\int_S fdM||^p)^{1/p}, \tag{3.1}$$

where $p \in (0,\alpha)$. $\lambda_{\alpha,p}$ is a well-defined quasi norm on L_B^{simple}. Since $\int fdM$ is a B-valued SαS random vector such that

$$E \exp(i < \int fdM,x'>) = \exp\, (-\int |<f(s),x'>|^\alpha dm(s)), \tag{3.2}$$

$x' \in B'$, inequality (2.3) yields

$$\lambda_{\alpha,p}(f) \geq C(\int_S ||f(s)||^\alpha \, dm(s))^{1/\alpha} \tag{3.3}$$

where $C = C(\alpha,p)$. Moreover, by (2.2) $\lambda_{\alpha,p} \underset{C}{\sim} \lambda_{\alpha,q}$ for every $p,q \in (0,\alpha)$, where $C=C(\alpha,p,q)$.

Let S_B^α be a completion of L_B^{simple} in $\lambda_{\alpha,p}$. In view of (3.3) S_B^α can be realized as a linear subspace of $L_B^\alpha = L_B(S,\sigma(A),m)$ as follows:

$$S_B^\alpha = \{f \in L_B^\alpha\text{: there exists } \{f_n\}_{n=1}^\infty \subset L_B^{simple} \text{ such that } f_n \to f \text{ in } L_B^\alpha \text{ and}$$

$$\lim_{n,m\to\infty} \lambda_{\alpha,p}(f_n - f_m) = 0 \text{ for some (each) } p \in (0,\alpha)\}.$$

By (3.1) the mapping $f \to \int_S fdM$ extends to an isometric injection of $(S_B^\alpha,\lambda_{\alpha,p})$

into $L^p_B(\Omega,F,P)$. Values of this extension are also denoted by $\int f dM$ and called the stochastic integral of f with respect to M.

Because of the lack of (2.4) in general, the stable stochastic integral can not be defined for all f's in $L^\alpha_B(S,\sigma(A),m)$ and S^α_B is the largest subspace of L^α_B where $\int f dM$ is defined by taking limits of stochastic integrals of simple functions. Although the dependence of $\lambda_{\alpha,p}(f)$ on f is not given explicitly, this is a useful quasi-norm which can be effectively estimated in many concrete examples of B (see e.g. Sections 6 and 7).

We shall frequently use the following particular case of Ito-Nisio theorem for Banach space valued stochastic integrals which was proven in [26].

<u>Proposition 3.1</u>. $f \in S^\alpha_B$ *and* $\mu = L(\int f dM)$ *if and only if* $\int |\langle f(s),x'\rangle|^\alpha dm(s) < \infty$ *for every* $x' \in B'$ *and the cylindrical measure* μ_0 *given by*

$$\hat{\mu}_0(x') = \exp\{-\int |\langle f(s),x'\rangle|^\alpha dm(s)\}, \quad x' \in B',$$

extends to a countably additive Borel measure μ *on* B.

As a consequence of the above proposition every SαS p.m. on B has a stochastic integral representation which follows from the following (cf. [26], Theorem 6.7):

<u>Proposition 3.2</u>. $\{\int f dM: f \in S^\alpha_B\}$ *is a closed linear subspace of* $L^p_B(\Omega,F,P)$ *consisting of* SαS *random vectors. If* (S,A,m) *is atomless, then* $\{L(\int f dM): f \in S^\alpha_B\}$ *coincides with the set of all* SαS *p.m.'s on* B.

We shall discuss now a different transformation of f which leads to the same distribution as $\int f dM$. We shall consider a series representation of stable random vectors due to LePage, Woodroofe and Zinn [13] as it was developed by Marcus and Pisier [17].

Assume that $m(S) < \infty$ and $A = \sigma(A)$. Let $\{\eta_j\}$ be a sequence of positive i.i.d. random variables such that $P(\eta_n > \lambda) = e^{-\lambda}$, $\lambda \geq 0$, and put $\Gamma_j = \eta_1 + \ldots + \eta_j$. Let $\{\xi_j\}$ be an i.i.d. sequence of symmetric random variables such that $E|\xi_j|^\alpha = 1$. Let $\{\tau_j\}$ be a sequence of independent uniformly distributed random elements in (S,A,m), i.e. $P(\tau_j \in A) = m(A)/m(S)$ for every $A \in A$. We assume that all the sequences $\{\eta_j\}$, $\{\xi_j\}$ and $\{\tau_j\}$ are independent of the others.

Let $f \in S_B^\alpha$ and $\mu = L(\int f dM)$. Then for every $x' \in E'$

$$E|\langle f(\tau_j),x'\rangle|^\alpha = [m(S)]^{-1} \int_S |\langle f(s),x'\rangle|^\alpha \, dm(s) < \infty$$

and by Lemma 1.4 in [17] the series

$$c(\alpha)[m(S)]^{1/\alpha} \sum_{j=1}^{\infty} (\Gamma_j)^{-1/\alpha} \xi_j \langle f(\tau_j),x'\rangle$$

converges a.s. to a real SαS random variable with the characteristic function $\phi(t) = \exp(-|t|^\alpha \int |\langle f(s),x'\rangle|^\alpha \, dm(s))$, where $c(\alpha) = (\int_0^\infty x^{-\alpha} \sin x dx)^{-1/\alpha}$. Since $\phi(t) = \hat{\mu}(tx')$ and the sequence $\{\Gamma_j \xi_j f(\tau_j)\}$ is sign-invariant, *Ito-Nisio* theorem (see e.g. [16], II.4.3. and II.4.4) yields the a.s. convergence of the series

$$\Sigma(f) = c(\alpha)[m(S)]^{1/\alpha} \sum_{j=1}^{\infty} (\Gamma_j)^{-1/\alpha} \xi_j f(\tau_j) \tag{3.4}$$

and $L(\Sigma(f)) = \mu$. Conversely, if (3.4) converges a.s. or in P, then the function $x' \to \exp(-\int|\langle f,x'\rangle|^\alpha dm)$ is the characteristic functional of $\Sigma(f)$ and by Proposition 3.1. $f \in S_B^\alpha$.

We summarize above in the following:

<u>Proposition 3.3</u>. *Let* $m(S) < \infty$ *and* $A = \sigma(A)$. *Then* $f \in S_B^\alpha$ *if and only if* $\Sigma(f)$ *converges a.s. or in* P. *Moreover,*

$$L(\int f dM) = L(\Sigma(f))$$

4. Bounds for moments of an SαS stochastic integral.

To obtain the first proposition we argue similarly to Marcus [15] and to Giné et al. [7].

Since $j^{-1}\Gamma_j \to 0$ a.s. by Kolmogorov's SLLN, we get that $\Sigma(f)$ converges a.s. if and only if

$$A(f) = \sum_{j=1}^{\infty} j^{-1/\alpha}\xi_j f(\tau_j) \tag{4.1}$$

converges a.s. (see (3.4)). Moreover, by contraction principle we have

$$2^{-1/p}[E \inf_j (j/\Gamma_j)^{p/\alpha}]^{1/p}[m(S)]^{1/\alpha}(E||A(f)||^p)^{1/p} \leq$$

$$\leq (E||\Sigma(f)||^p)^{1/p} \leq$$

$$2^{1/p}[E \sup_j (j/\Gamma_j)^{p/\alpha}]^{1/p}[m(S)]^{1/\alpha}(E||A(f)||^p)^{1/p},$$

where $C = C(\alpha)$ and $p \in (0,\alpha)$. Since $E \inf_j (j/\Gamma_j)^{p/\alpha} > 0$ and $E \sup_j (j/\Gamma_j)^{p/\alpha} < \infty$

(see [15] and [7] respectively) we get

Proposition 4.1. *Let* $m(S) < \infty$ *and* $A=\sigma(A)$. Then $f \in S_B^\alpha$ *if and only if* $A(f)$ *converges a.s. and/or in* L_B^p *for some (each)* $p \in [0,\alpha)$. Moreover,

$$(E||\int f dM||^p)^{1/p} \underset{C}{\approx} [m(S)]^{1/\alpha} (E||A(f)||^p)^{1/p}$$

where $C = C(\alpha,p)$, $p \in (0,\alpha)$.

Let us note that even on the real line $A(f)$ need not have the α-th moment finite. This is a simple corollary to Proposition 4.2 in [5]. We shall normalize f to ensure finiteness of all moments of $A(f)$.

Let x_0 be a fixed point on the unit sphere of B. For every $f \in L_B^\alpha$ we define $\overline{f}(s) = f(s)/||f(s)||$ if $f(s) \neq 0$ and $\overline{f}(s) = x_0$ otherwise. We define also a finite measure m_f on $(S,\sigma(A))$ by $dm_f(s) = ||f(s)||^\alpha dm(s)$ ($m(S)$ can be infinite). Let M_f be an $S\alpha S$ random measure on $(S,\sigma(A))$ with the control measure m_f. In view of Proposition 3.1 $f \in S_B(S,A,m)$ if and only if $f \in L_B^\alpha$ and $\overline{f} \in S_B^\alpha(S,\sigma(A),m_f)$. Moreover

$$(4.2) \qquad \mathcal{L}\left(\int_S f dM\right) = \mathcal{L}\left(\int_S \overline{f} dM_f\right).$$

Let $\{\tau_j^f\}$ be a sequence of independent uniformly distributed random elements in $(S,\sigma(A),m_f)$ i.e. $P\{\tau_j^f \in A\} = m_f(A)/m_f(S)$. Let $\{\varepsilon_j\}$ be a sequence of i.i.d. random variables such that $P\{\varepsilon_j = -1\} = P\{\varepsilon_j = 1\} = 1/2$ and independent of $\{\tau_j^f\}$. By Proposition 4.1 we get

$$(E\, ||\int \overline{f} dM_f\, ||^p)^{1/p} \underset{C}{\sim} \left(\int ||f||^\alpha\, dm\right)^{1/\alpha} (E||A(\overline{f})||^p)^{1/p},$$

where $C = C(\alpha,p)$ and $A(\overline{f}) = \sum_{j=1}^\infty j^{-1/\alpha}\varepsilon_j \overline{f}(\tau_j^f)$. Since $\sup_j ||j^{-1/\alpha}\varepsilon_j\overline{f}(\tau_j^f)|| = 1$ $A(\overline{f})$ converges in L_B^r for every $r > 0$. Using Theorem 3.3 in Giné and Zinn [8] we obtain for every $p \in (0,\alpha)$ and $r > 0$

$$(E||A(\overline{f})||^p)^{1/p} \underset{C}{\sim} 1 + \left(E\, ||\sum_{j=j_0+1}^\infty j^{-1/\alpha}\varepsilon_j\overline{f}(\tau_j^f)||^r\right)^{1/r},$$

where $C = C(p,r)$ and j_0 is the greatest integer not exceeding $8^{-1}3^{p\vee r}$. By contraction principle we have

$$E\, ||\sum_{j=j_0+1}^\infty j^{-1/\alpha}\varepsilon_j\overline{f}(\tau_j^f)||^r = E\, ||\sum_{i=1}^\infty (j_0+i)^{-1/\alpha}\varepsilon_i\overline{f}(\tau_i^f)||^r$$

$$\geq 2^{-1}(1 + j_0)^{-r/\alpha}\, E||A(\overline{f})||^r$$

and clearly

$$E\,\|\sum_{j=j_0+1}^{\infty} j^{-1/\alpha}\varepsilon_j\,\overline{f}(\tau_j^f)\|^r \le 2\,E\,\|A(\overline{f})\|^r.$$

We have proven the following

Theorem 4.2. $f \in S_B^\alpha$ *if and only if* $f \in L_B^\alpha$ *and the series* $\sum_{j=1}^{\infty} j^{-1/\alpha}\varepsilon_j\overline{f}(\tau_j^f)$ *converges in* L_B^r *for some (each)* $r \ge 0$. *Moreover,*

$$(E\,\|\int f dM\|^p)^{1/p} \underset{C}{\sim} (\int \|f\|^\alpha dm)^{1/\alpha}[1+(E\|\sum_{j=1}^{\infty} j^{-1/\alpha}\varepsilon_j\overline{f}(\tau_j^f)\|^r)^{1/r}]$$

where $C = C(\alpha,p,r)$, $p \in (0,\alpha)$ *and* $r > 0$.

We shall study now the relationship between boundedness and convergence in (4.1). In view of Proposition 4.1 this will give us an additional information about S_B^α. Let $m(S) < \infty$, $A = \sigma(A)$ and let

$$bS_B^\alpha = bS_B^\alpha(S,A,m) = \{f\colon S \to B\colon \sup_n \|\sum_{j=1}^{n} j^{-1/\alpha}\xi_j f(\tau_j)\| < \infty \text{ a.s.}\}.$$

According to Proposition 4.1

$$S_B^\alpha = S_B^\alpha(S,A,m) = \{f\colon S \to B\colon \sum_{j=1}^{\infty} j^{-1/\alpha}\xi_j f(\tau_j) \text{ conv. a.s.}\}.$$

Obviously $S_B^\alpha \subset bS_B^\alpha$. Let

$$\|f\|_{\alpha,p} = \sup_n (E\,\|\sum_{j=1}^{n} j^{-1/\alpha}\,\xi_j f(\tau_j)\|^p)^{1/p} \quad \text{for } p \in (0,\alpha),$$

and let

$$||f||_{\alpha,0} = \sup_n E\,(||\sum_{j=1}^{n} j^{-1/\alpha}\,\xi_j\, f(\tau_j)|| \wedge 1).$$

It is standard to check that $||\ ||_{\alpha,0}$ is a complete F-norm on bS_B^α as well as on S_B^α. Moreover, by Proposition 4.1 all the F-norms $||\ ||_{\alpha,p}$ are equivalent on S_B^α, $p \in [0,\alpha)$.

Lemma 4.3. *For every* $f \in bS_B^\alpha$ *and* $p \in (0,\alpha)$ $||f||_{\alpha,p} < \infty$. *Moreover* $bS_B^\alpha \subset L_B^\alpha$.

Proof. Without loss of generality we may assume that $B = C[0,1]$. Let $\{P_k\}$ be a sequence of finite rank operators on B with $||P_k|| \leq 1$ and such that $P_k x \to x$ for every $x \in B$. Put $f_k(s) = P_k f(s)$, $s \in S$. Clearly $\sup_n ||\sum_{j=1}^{n} j^{-1/\alpha}\,\varepsilon_j f_k(\tau_j)\,|| < \infty$ a.s. for every k and since $\dim P_k B < \infty$ the series $\sum_{j=1}^{\infty} j^{-1/\alpha}\,\xi_j\, f_k(\tau_j)$ converges a.s. By Proposition 4.1 $f_k \in S_B^\alpha$. Since $||\ ||_{\alpha,0}$ and $||\ ||_{\alpha,p}$ are equivalent on S_B^α there exists $\varepsilon > 0$ such that $||g||_{\alpha,p} \leq 1$ for every $g \in S_B^\alpha$ with $||g||_{\alpha,0} \leq \varepsilon$. Since $f \in bS_B^\alpha$ there exists $\delta > 0$ such that $||\delta f||_{\alpha,0} \leq \varepsilon$. Therefore $||\delta f_k||_{\alpha,0} \leq \varepsilon$ for every k and because $f_k \in S_B^\alpha$ $||\delta f_k||_{\alpha,p} \leq 1$. By (3.3) and Proposition 4.1 $(\int ||\delta f_k||^\alpha dm)^{1/\alpha} \leq$ $\leq$ const $||\delta f_k||_{\alpha,p} \leq$ const. Letting $k \to \infty$ we get $||f||_{\alpha,p} \leq \delta^{-1} < \infty$ as well as $\int ||f||^\alpha dm < \infty$.

Corollary 4.4. $S_B^\alpha \subset bS_B^\alpha \subset L_B^\alpha$. *For every* $p \in [0,\alpha)$ $||\ ||_{\alpha,p}$ *is a complete F-norm on* bS_B^α *as well as on* S_B^α.

Proposition 4.1 and above Corollary give the following:

Theorem 4.5. *S_B^α is the smallest closed subspace of bS_B^α containing all simple functions. In other words $f \in S_B^\alpha$ if and only if for every $\varepsilon > 0$ and $p < \alpha$ there exists a simple function f_ε such that*

$$E\|\sum_{j=1}^{n} j^{-1/\alpha}\xi_j(f - f_\varepsilon)(\tau_j)\|^p < \varepsilon$$

for all $n \in \mathbb{N}$

Remarks:

(a) For the sufficiency f_ε need not be a simple function. It is enough to have $f_\varepsilon \in S_B^\alpha$.

(b) $bS_B^\alpha = S_B^\alpha$ provided B does not contain a subspace isomorphic to c_0.

(c) $bS_B^\alpha = S_B^\alpha = L_B^\alpha$ provided that B is of stable type α (cf. [18] and Lemma 4.3). In particular these equalities hold for $\alpha < 1$ and any Banach space B.

(d) In general $S_B^\alpha \not\subseteq bS_B^\alpha \not\subseteq L_B^\alpha$. Indeed, Sztencel [31] has showed that for every $\alpha \geq 1$ there exists a Banach space B and a sequence $\{x_n\} \subset B$ such that $\sup_n E\|\Sigma_{j=1}^n \theta_j x_j\|^p < \infty$ for every $p < \alpha$ and $\Sigma_{j=1}^\infty \theta_j x_j$ diverges a.s., where $\{\theta_j\}$ is a sequence of i.i.d. random variables with $E\exp(it\theta_1) = \exp(-|t|^\alpha)$. Let M be an SαS random measure on Borel subsets of the unit interval and with the Lebesgue measure as the control measure. Put $f_n = n^{1/\alpha}\Sigma_{j=1}^n x_j I_{[(j-1)/n,j/n]}$. Since $L(\int_0^1 f_n dM) = L(\Sigma_{j=1}^n \theta_j x_j)$, $\{f_n\}$ is a bounded sequence in S_B^α. Therefore $\sup_n \int_0^1 \|f_n\|^\alpha dt < \infty$. This yields $c^{-1} = \Sigma_{j=1}^\infty \|x_j\|^\alpha < \infty$. Let $\{A_j\}$ be a partition of $[0,1]$ such that $|A_j| = c\|x_j\|^\alpha$ and define $g_k = \Sigma_{j=1}^k (x_j/\|x_j\|) I_{A_j}$ and $g = \lim_{k\to\infty} g_k$. Since $L(\int_0^1 g_k dM) = L(c^{1/\alpha}\Sigma_{j=1}^k \theta_j x_j)$, by Proposition 3.1 and Ito-Nisio theorem $g \notin S_B^\alpha$. On the other hand $g_k \in S_B^\alpha$ and by Proposition 4.1 for every n and $p < \alpha$

$$E\,||\textstyle\sum_{j=1}^{n} j^{-1/\alpha}\xi_j g(\tau_j)||^p \le \liminf_{k\to\infty} E\,||\sum_{j=1}^{n} j^{-1/\alpha}\xi_j g_k(\tau_j)||^p$$

$$\le \liminf_{k\to\infty} ||g_k||_{\alpha,p}^p \le C \liminf_{k\to\infty} |E\,||\int_0^1 g_k dM||^p|$$

$$\le C\, c^{p/\alpha} \liminf_{k\to\infty} E\,||\sum_{j=1}^{k} x_j\theta_j||^p$$

$$\le \text{Const.}$$

This proves that $g \in bS_B^\alpha$.

To show that $bS_B^\alpha \not\subset L_B^\alpha$ (in general) it is enough to choose $\alpha \geq 1$ and Banach space B which is not of stable type α and does not contain any subspace isomorphic to c_0. Then by (b) $bS_B^\alpha = S_B^\alpha$ and $S_B^\alpha \not\subset L_B^\alpha$ (cf. [18]).

5. Modification of the kernel of a stochastic integral process.

In this section we shall study processes $X(t)$, $t \in T$ which sample paths $X(\cdot,\omega)$ belong to a separable Banach space $V(T)$ of functions defined on T. Let C_T be the cylindrical σ-field of $V(T)$ i.e. the smallest σ-field of subsets of $V(T)$ such that all evaluations: δ_t: $V(T) \to \mathbb{R}$, where $\langle x,\delta_t\rangle = x(t)$, $x \in V(T)$, $t \in T$, are measurable. The equality

$$C_T = \text{Borel } (V(T)) \tag{5.1}$$

is necessary and sufficient for regarding stochastic processes with sample paths in $V(T)$ as Borel measurable random elements in $V(T)$. Observe that the inclusion $C_T \subset$ Borel $(V(T))$ implies that every evaluation δ_t is Borel measurable, and since δ_t is linear, Banach theorem yields that δ_t is continuous. Conversely, if all evaluations δ_t, $t \in T$ are continuous, then (5.1) holds. Indeed, since evaluations separate points in $V(T)$ one can easily deduce from Hahn-Banach theorem (see e.g. [24], Sec.2, Chap.2) that the set $W=\{\Sigma_{j=1}^n a_j\delta_{t_j} : a_j \in \mathbb{R}, t_j \in T, n \geq 1\}$ is dense in $[V(T)]'$ with respect to the weak-star topology. Since $V(T)$ is separable, W is also sequentially weak-star dense in $[V(T)]'$, and consequently, every functional $x' \in [V(T)]'$ is C_T-measurable. Again by separability of $V(T)$ we get that Borel $(V(T)) \subset C_T$. Therefore (5.1) is equivalent to the assumption that all evaluations $x \to x(t)$ are continuous.

Theorem 5.1. *Let $V(T)$ be a separable Banach space of functions defined on T such that all evaluations $x \to x(t)$ are continuous. Assume that the $S\alpha S$ stochastic process*

$$X(t) = \int_S h(t,s)\, dM(s), \quad t \in T,$$

has a modification X_0 *with sample paths in* $V(T)$, *where* $h\colon T\times S \to \mathbb{R}$ *is such that* $h(t,\cdot) \in L^{\alpha}(S,\delta(A),m)$ *for every* $t \in T$.

Then there exists a function $h_0\colon T\times S \to \mathbb{R}$ *such that*

(i) *for every* $s \in S$ $h_0(\cdot,s) \in V(T)$;

(ii) *for every* $t \in T$ $h_0(t,\cdot) = h(t,\cdot)$ m-*almost everywhere on* S;

(iii) *for every* $x' \in [V(T)]'$

$$\langle X_0(\cdot,\omega),x'\rangle = \int_S \langle h_0(\cdot,s),x'\rangle \, dM(s)(\omega),$$

for almost all $\omega \in \Omega$.

Proof. The proof is divided in three parts.

Claim 1. Assume that T is a compact metric space and $V(T) = C(T)$ is the space of all continuous functions on T with the supremum norm. Then the conclusion of Theorem 5.1 is true.

Proof of claim 1. Let D be a finite subset of $T\times T$. First we shall show that

$$\int_S \max\{|h(t_1,s) - h(t_2,s)|^{\alpha}\colon\ (t_1,t_2) \in D\}\, dm(s) \tag{5.2}$$

$$\le C(E\max\{|X(t_1) - X(t_2)|^p\colon (t_1,t_2) \in D\})^{\alpha/p},$$

where $C = C(\alpha,p)$ and $p \in (0,\alpha)$.

Indeed, let us define an SαS random vector in $\mathbb{R}^D$ by

$$Y = \{(X(t_1) - X(t_2))\}_{(t_1,t_2)\in D}$$

and consider $\mathbb{R}^D$ as a Banach space with the norm $||a|| = \max\{|a(t_1,t_2)|\colon (t_1,t_2) \in D\}$. Then for every $b \in \mathbb{R}^D$ we have

$$E \exp(i \langle Y,b\rangle) = E \exp\{i\sum b(t_1,t_2)\,[X(t_1) - X(t_2)]\}$$

$$= E \exp\{i\int_S \sum b(t_1,t_2)\,[h(t_1,s) - h(t_2,s)]\,dM(s)\}$$

$$= \exp(-\int_S |\langle f(s),b\rangle|^\alpha\,dm(s)),$$

where $f:S \to \mathbb{R}^D$, $f(s) = \{(h(t_1,s) - h(t_2,s))\}_{(t_1,t_2)\in D}$. By (2.3) we get

$$\left(\int_S ||f(s)||^\alpha\,dm(s)\right)^{1/\alpha} \le C(E\,||Y||^p)^{1/p}$$

which yields (5.2).

Let d be a metric on T. For n=1,2,... let T_n be a finite 1/n - net in T and let $T_\infty = \bigcup_{n=1}^{\infty} T_n$. Clearly T_∞ is dense in T. Define for $x: T \to \mathbb{R}$, $\delta > 0$ and

$n\in\mathbb{N}$

$$\phi_{\delta,n}(x) = \max\{|x(t_1) - x(t_2)|: t_1,t_2\in T_n,\ d(t_1,t_2) < \delta\}$$

and

$$\phi_\delta(x) = \sup\{|x(t_1) - x(t_2)|: t_1,t_2 \in T_\infty,\ d(t_1,t_2) < \delta\}.$$

Inequality (5.2) applied for $D = \{(t_1,t_2) \in T_n \times T_n: d(t_1,t_2) < \delta\}$ yields

$$\int_S \{\phi_{\delta,n}\,[h(\cdot,s)]\}^\alpha\,dm(s) \le C[\int_\Omega \{\phi_{\delta,n}[X_0(\cdot,\omega)]\}^p dP(\omega)]^{\alpha/p}.$$

Since $\phi_{\delta,n}[X_0(\cdot,\omega)] \le 2 \sup_{t\in T} |X_0(t,\omega)| = 2\,||\,X_0(\cdot,\omega)\,||$ letting $n \to \infty$ we obtain

$$\int_S \{\phi_\delta[h(\cdot,s)]\}^\alpha\,dm(s) \le C[\int_\Omega \{\phi_\delta[X_0(\cdot,\omega)]\}^p dP(\omega)]^{\alpha/p}$$

and since sample paths of X_0 are continuous we get

$$(5.3) \qquad \lim_{\delta \downarrow 0} \phi_\delta[h(\cdot,s)] = 0$$

for m-almost all $s \in S$. Let $s \in S$. If (5.3) holds then $h(\cdot,s)$ is uniformly continuous on T_∞ and there is unique continuous function $h_0(\cdot,s)$ defined on T which is equal to $h(\cdot,s)$ on T_∞. If (5.3) fails, then we put $h_0(t,s) = 0$ for all $t \in T$. Therefore $h_0(\cdot,s) \in C(T)$ for all $s \in S$ and $h_0(t,\cdot) = h(t,\cdot)$ m-almost everywhere for every $t \in T_\infty$. By stochastic continuity of X and continuity of $t \to h_0(t,s)$ for every $s \in S$ we get (ii) for every $t \in T$. Clearly (iii) is satisfied for all x's of the form $x' = \sum_{j=1}^{n} a_j \delta_{t_j}$. Since such functionals are sequentially weak-star dense in $[V(T)]'$ (iii) follows.

Claim 2. Assume that $V(T)$ is a closed subspace of $C(T)$, where T is a compact metric space. Then the conclusion of Theorem 5.1 holds.

Proof of claim 2. By claim 1 there exists a function h_0 such that $h_0(\cdot,s) \in C(T)$, (ii) and (iii) hold ((iii) for every $x' \in [C(T)]'$). Let

$$[V(T)]^\perp = \{x' \in [C(T)]' : \langle x,x'\rangle = 0 \text{ for all } x \in V(T)\}.$$

For every $x' \in [V(T)]^\perp$ we have

$$\int_S \langle h_0(\cdot,s),x'\rangle dM(s) \overset{a.s.}{=} \langle X_0(\cdot,\cdot),x'\rangle = 0.$$

Therefore $\langle h_0(\cdot,s),x'\rangle = 0$ for m-almost every $s \in S$ and every $x' \in [V(T)]^\perp$. Let Γ be a countable weak-star dense subset of $[V(T)]^\perp$. Define

$$S_0 = \{s \in S : \langle h_0(\cdot,s),x'\rangle = 0 \text{ for all } x' \in \Gamma\}.$$

Then $m(S \setminus S_0) = 0$ and for every $s \in S_0$ $h_0(\cdot,s) \in V(T)$. Thus a function h_1

defined by $h_1(\cdot,s) = h_0(\cdot,s)$ for $s \in S_0$ and $h_1(\cdot,s) \equiv 0$ for $s \notin S_0$ fulfills (i), (ii) and (iii) of Theorem 2.1.

Proof of the theorem in general. Let $U' = \{x' \in [V(T)]' : ||x'|| \leq 1\}$. U' equipped with the relative weak-star topology is a metrizable compact space. Let Φ: $V(T) \to C(U')$ be defined by $[\Phi(x)]\,(x') = \langle x,x' \rangle$, $x \in V(T)$ and $x' \in U'$. It is easy to check that Φ is an isometric linear injection of $V(T)$ into $C(U')$. Put $V(U') = \Phi[V(T)]$. Since the mapping $\Omega \ni \omega \to X_0(\cdot,\omega) \in V(T)$ is Borel measurable we obtain that Y: $\Omega \to V(U')$ defined by $Y(\cdot,\omega) = \Phi[X_0(\cdot,\omega)]$ is also Borel measurable. Therefore $Y(x',\cdot)$, $x' \in U'$ is a stochastic process with continuous sample paths belonging to $V(U')$. Let W be the set of all linear combinations of δ_t's. By the discussion preceeding Theorem 5.1, for every $x' \in U'$ there exists $\{x_n'\} \subset W \cap U'$, $x_n' = \Sigma_{j=1}^{k_n} a_{nj}\delta_{t_{nj}}$, such that $\langle x,x_n' \rangle \to \langle x,x' \rangle$ for every $x \in V(T)$. Therefore

$$\begin{aligned} Y(x',\cdot) &= \lim_{n\to\infty} Y(x_n',\cdot) = \lim_{n\to\infty} \langle X_0(\cdot,\cdot),x_n' \rangle \\ &= \lim_{n\to\infty} \sum_{j=1}^{k_n} a_{nj}X(t_{nj}) \\ &= \lim_{n\to\infty} \int_S [\sum_{j=1}^{k_n} a_{nj}h(t_{nj},s)]\, dM(s), \end{aligned}$$

a.s., where the first equality holds point-wise by continuity of sample paths of Y. Thus $\Sigma_{j=1}^{k_n} a_{nj}h(t_{nj},\cdot)$ converge in $L^\alpha(S,\sigma(A),m)$ to some function $g(x',\cdot)$ and we have

$$Y(x',\cdot) = \int_S g(x',s)\, dM(s) \quad \text{a.s.}$$

for every $x' \in U'$. Moreover $g(a\delta_t,\cdot) = ah(t,\cdot)$ m-almost everywhere on S, provided $a\delta_t \in U'$. According to claim 2 there exists $g_0(x',s)$ such that $g_0(\cdot,s) \in V(U')$ for all $s \in S$ and $g_0(x',\cdot) = g(x',\cdot)$ m-almost everywhere for every $x' \in U'$.

Define a Borel measurable function $G: S \to V(U')$ by $[G(s)](x') = g_0(x',s)$ and $H:S \to V(T)$ by $H = \Phi^{-1}\circ G$. Let $h_0(t,s) = \langle H(s),\delta_t\rangle$. Clearly (i) is fulfilled. To show (ii) let $t \in T$ and let $a > 0$ *be such that* $a\delta_t \in U'$. *Since* $H(s) = \Phi^{-1}[G(s)]$ if and only if $\langle H(s),x'\rangle = g_0(x',s)$ for all $x' \in U'$ we obtain

$$ah_0(t,s) = \langle H(s),a\delta_t\rangle = g_0(a\delta_t,s) = g(a\delta_t,s) = ah(t,s),$$

where the last two equalities hold for m-almost all $s \in S$. (ii) is proved. (iii) follows by the weak-star density of W in $[V(T)]'$. The proof of Theorem 5.1 is complete.

<u>Corollary 5.2</u>. *Let* X_0 *and* h_0 *be as in Theorem* 5.1. *Then the function* $f: S \to V(T)$ *defined by* $[f(s)](t) = h_0(t,s)$ *belong to* $S^{\alpha}_{V(T)}$. *In particular,* $\int_S ||h_0(\cdot,s)||^{\alpha}_{V(T)}dm(s) < \infty$. *Further, for every* $\varepsilon > 0$ *and* $p \in (0,\alpha)$ *there exist a finite sequence* $\{x_j\}_{j=1} \subset V(T)$ *and pairwise disjoint sets* $A_1,\dots,A_n \in A$ *such that*

$$E||X_0(\cdot) - X_\varepsilon(\cdot)||^p_{V(T)} < \varepsilon$$

where $X_\varepsilon(t) = \Sigma^n_{j=1}x_j(t)M(A_j)$, $t \in T$.

<u>Proof</u>. Follows by (iii) of Theorem 5.1 and Proposition 3.1.

6. A characterization of SαS processes with absolute continuous trajectories.

A characterization of SαS processes ($1 < \alpha < 2$) with absolute continuous sample paths in terms the so-called covariation function has been obtained by Cambanis and Miller in [3]. We shall characterize SαS processes with above sample path property using the representing function h. Moreover, in our case $0 < \alpha < 2$.

Let us recall that a function $x\colon [a,b] \to \mathbb{R}$ is absolutely continuous if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that for every disjoint family $\{(t_k,u_k)\}_{k=1}^{n}$ of subintervals of $[a,b]$ $\Sigma_{k=1}^{n}|x(u_k) - x(t_k)| < \varepsilon$, provided $\Sigma_{k=1}^{n}|u_k - t_k| < \delta$. Then x is absolutely continuous if and only if there exists $\dot{x} \in L^1[a,b]$ such that

$$x(u) = x(a) + \int_a^u \dot{x}(t)dt$$

for every $a \le u \le b$. Every absolutely continuous function x is differentiable almost everywhere and $\frac{dx}{dt} = \dot{x}$ almost everywhere on $[a,b]$. Let $AC^p\,[a,b]$ be the space of all absolutely continuous functions on $[a,b]$ whose derivatives are integrable in the p-th power, $p \ge 1$.

Let

$$X(t) = \int_S h(t,s)dM(s), \qquad t \in [a,b]$$

be an SαS process, where M is an SαS random measure defined on a δ-ring of subsets of S and with the control measure m. As before h is a deterministic function such that $h(t,\cdot) \in L^\alpha(S,\sigma(A),m)$ for every $t \in [a,b]$.

Theorem 6.1. X *has a modification with sample paths in* $AC^p[a,b]$ *if and* *only if* h *admits a modification* h_o *such that*

(i) *for every* $t \in [a,b]$ $h_o(t,\cdot) = h(t,\cdot)$ m-*almost everywhere*,

(ii) *for every* $s \in S$ $h_o(\cdot,s) \in AC^p[a,b]$,

(iii) $\Omega_{\alpha,p}(\frac{\partial h_o}{\partial t}) < \infty$,

where

$$\Omega_{\alpha,p}(g) = \begin{cases} \int_S (\int_a^b |g(t,s)|^p dt)^{\alpha/p} dm(s) & \text{if } 0 < \alpha < p, \\ \int_S \int_a^b |g(t,s)|^\alpha \left[1+\log_+ \dfrac{|g(t,s)|^\alpha \int_S \int_a^b |g(u,v)|^\alpha du\, dm(v)}{\int_a^b |g(u,s)|^\alpha du \int_S |g(t,v)|^\alpha dm(v)}\right] dt\, dm(s), & \text{if } \alpha = p, \\ \int_a^b (\int_S |g(t,s)|^\alpha dm(s))^{p/\alpha} dt & \text{if } 1 \le p < \alpha. \end{cases}$$

The proof of Theorem 6.1 is preceded by the lemma below which extends Proposition 4.2 in [5], where only the case $p = \alpha$ has been considered. Since we use a similar argument in all cases of p, the case $p = \alpha$ is also proven here.

Lemma 6.2. *Let* $\xi, \xi_1, \xi_2, \ldots$ *be i.i.d. symmetric random variables and* $0 < \alpha < 2$. *Then the series* $\sum_j j^{-1/\alpha}\xi_j$ *converges a.s. if and only if* $E|\xi|^\alpha < \infty$. *Further, for every* $p > 0$

$$(E|\sum_j j^{-1/\alpha}\xi_j|^p)^{1/p} \le C \begin{cases} (E|\xi|^p)^{1/p} & \text{if } \alpha < p \\ [E|\xi|^\alpha (1+\log_+ \dfrac{|\xi|^\alpha}{E|\xi|^\alpha})]^{1/\alpha} & \text{if } \alpha = p \\ (E|\xi|^\alpha)^{1/\alpha} & \text{if } p < \alpha \end{cases}$$

where $C = C(\alpha,p)$.

Proof. Since $L(\xi_j) = L(\xi)$ we have for every $t > 0$

(6.1) $$t^{-\alpha}E\,|\xi|^{\alpha} - 1 \le \sum_j P(\,|\xi_j| > tj^{1/\alpha}) \le t^{-\alpha}\,E\,|\xi|^{\alpha}.$$

Therefore $E\,|\xi|^{\alpha} < \infty$ is a necessary condition for the a.s. convergence of $\sum_j j^{-1/\alpha}\,\xi_j$. The sufficiency follows from (6.1) and the following estimates for every $t>0$

$$\sum_j E(j^{-1/\alpha}\,\xi_j I(|\xi_j| \le tj^{1/\alpha}))^2 = \sum_j j^{-2/\alpha}\,E\,|\xi|^2\,I(|\xi| \le tj^{1/\alpha})$$

$$= E\,|\xi|^2 \sum_{j \ge |\xi/t|^{\alpha}} j^{-2/\alpha}$$

(6.2)

$$\le E|\xi|^2\,[(\left|\frac{\xi}{t}\right|^{\alpha})^{-2/\alpha} + (\left|\frac{\xi}{t}\right|^{\alpha})^{1-2/\alpha}]$$

$$= t^2\,(1 + \frac{\alpha}{2-\alpha}\,E\,\left|\frac{\xi}{t}\right|^{\alpha}).$$

To estimate the moments of $\sum j^{-1/\alpha}\xi_j$ we use Corollary 3.4 in Gine and Zinn [8] which gives

(6.3) $$E|\sum_j j^{-1/\alpha}\xi_j|^p \underset{C}{\sim} E\,\sup_j\,|j^{-1/\alpha}\xi_j|^p + [\sum_j E(j^{-1/\alpha}\xi_j I(|\xi_j| \le \delta j^{1/\alpha}))^2]^{p/2},$$

where $\delta = \inf\{t > 0\colon \Sigma\,P(|\xi| > tj^{1/\alpha}) \le 8^{-1}3^{2vp}\}$ and $C = C(p)$, $p>0$.
By (6.1) we get

(6.4) $$\delta \underset{C}{\sim} (E\,|\xi|^{\alpha})^{1/\alpha} \qquad \text{with } C = C(\alpha,p).$$

To obtain bounds for the first term on the right side of (6.3) we utilize

Lemma 3.2 in [8] which yields

(6.5) $$E \sup_j |j^{-1/\alpha}\xi_j|^p \underset{C}{\sim} \delta^p + R_{\alpha,p},$$

where C = C(p) and

$$R_{\alpha,p} = \sum_j E|j^{-1/\alpha}\xi_j I\,(\,|\xi_j| > \delta j^{1/\alpha})|^p$$

$$= E\,|\xi|^p \sum_{j<|\xi/\delta|^\alpha} j^{-p/\alpha}.$$

Since for every $x > 0$ $(1-r)^{-1}(x^{1-r}-1) \le \sum_{j<x} j^{-r} \le 1 + (1-r)^{-1}(x^{1-r}-1)$

provided $r \neq 1$ and $r > 0$ we get for $r = p/\alpha \neq 1$

$$R_{\alpha,p} \le E|\xi|^p\{1 + (1 - p/\alpha)^{-1}[(\left|\frac{\xi}{\delta}\right|^\alpha)^{1-p/\alpha} - 1]\}$$

$$\le \frac{p}{|\alpha - p|} E|\xi|^p + \frac{\alpha}{|\alpha - p|}\delta^{p-\alpha} E|\xi|^\alpha$$

and

$$R_{\alpha,p} \ge \frac{\alpha}{|\alpha - p|} E\,|\xi|^p - \frac{\alpha}{|\alpha - p|}\delta^{p-\alpha} E|\xi|^\alpha$$

Above estimates in conjunction with (6.4) and (6.5) give

(6.6) $$E \sup_j |j^{-1/\alpha}\xi_j|^p \underset{C}{\sim} E|\xi|^p + (E\,|\xi|^\alpha)^{p/\alpha}$$

with $C = C(\alpha,p)$ and $p \neq \alpha$. In the case $p = \alpha$ elementary inequalities $\log_+ x \le \sum_{j<x} j^{-1} \le 1 + \log_+ x$, $x > 0$ yield

$$E\,|\xi|^\alpha \log_+ \left|\frac{\xi}{\delta}\right|^\alpha \le R_{\alpha,\alpha} \le E|\xi|^\alpha \left(1 + \log_+\left|\frac{\xi}{\delta}\right|^\alpha\right).$$

Using (6.4) and (6.5) we get

$$(6.7) \qquad E \sup_j |j^{-1/\alpha}\xi_j|^\alpha \underset{C}{\sim} E\,|\xi|^\alpha \left(1 + \log_+ \frac{|\xi|^\alpha}{E\,|\xi|^\alpha}\right),$$

where $C = C(\alpha)$.

By (6.2) for $t = \delta$ and (6.4) the second term on the right side of (6.3) is bounded from above by $C(E\,|\xi|^\alpha)^{1/\alpha}$ with $C = C(\alpha,p)$. Therefore (6.6) for $p \neq \alpha$ and (6.7) for $p = \alpha$, respectively, in conjunction with (6.3) conclude Lemma 6.2.

Proof of Theorem 6.1. Clearly $AC^p[a,b]$ is a separable Banach space with the norm

$$||x|| = |x(a)| + \left(\int_a^b \left|\frac{dx}{dt}\right|^p dt\right)^{1/p},$$

and the evaluations $x \to x(t)$ are continuous for every $t \in [a,b]$. By Theorem 5.1 there exists a function h_0 satisfying (i) and (ii) provided X has a modification with sample paths in $AC^p[a,b]$. Then a function $f: S \to AC^p[a,b]$ defined by $[f(s)](t) = h_0(t,s)$ is Borel measurable and in view of Theorem 5.1 (iii) and Proposition 3.1 $f \in S^\alpha_{AC^p[a,b]}$. Conversely, if $f \in S^\alpha_{AC^p[a,b]}$ then $\int_S f(s)dM(s)$ is a random element in $AC^p[a,b]$ such that $X_0(t) = \langle \int_S f dM, \delta_t \rangle = \int_S h_0(t,s)dM(s) = X(t)$ a.s., i.e. X_0 gives a required modification of X. Therefore X has a modification with sample paths in $AC^p[a,b]$ if and only if (i), (ii) and $f \in S^\alpha_{AC^p[a,b]}$. Note also that without loss of generality we may assume that $h_0(a,\cdot) = 0$ (replacing X by $X_a(t) = X(t) - X(a)$).

By Theorem 4.2 $f \in S^\alpha_{AC^p[a,b]}$ if and only if $f \in L^\alpha_{AC^p[a,b]}$ and the series

$\Sigma j^{-1/\alpha}\varepsilon_j\bar{f}\,(\tau_j^f)$ converges in $L^q_{AC^p[a,b]}$ for some (each) $q \geq 0$. Here $[\bar{f}(s)](t) = h_o(t,s)||f(s)||^{-1}$ and

$$||f(s)|| = |h_o(a,s)| + \left(\int_a^b \left|\frac{\partial h_o}{\partial t}\right| (t,s)|^p dt\right)^{1/p} = \left(\int_a^b |g(t,s)|^p dt\right)^{1/p},$$

where $g(t,s) = \frac{\partial h_o}{\partial t}(t,s)$. Moreover τ_j^f 's are i.i.d. random elements in S such $P(\tau_j^f \in A) = m_f(A)/m_f(S)$, $A \in \sigma(A)$ and $dm_f(s) = ||f(s)||^\alpha dm(s)$. We have

$$(6.8) \qquad m_f(S) = \int_S ||f(s)||^\alpha\, dm(s) = \int_S \left(\int_a^b |g(t,s)|^p dt\right)^{\alpha/p} dm(s)$$

and $m_f(S) < \infty$ provided $f \in S^\alpha_{AC^p[a,b]}$. Further,

$$E\,||\sum_{j=1}^n j^{-1/\alpha}\varepsilon_j\bar{f}(\tau_j^f)||^p = \int_a^b E\,|\sum_{j=1}^n j^{-1/\alpha}\varepsilon_j\, g(t,\tau_j^f)||f(\tau_j^f)||^{-1}|^p dt = \int_a^b u_n(t)dt.$$

Since $\{u_n\}_{n=1}^\infty$ is a point-wise increasing sequence of functions and for every t

$$u_n(t) \geq E|\sum_{j=n_o}^n j^{-1/\alpha}\varepsilon_j g(t,\tau_j^f)||f(\tau_j^f)||^{-1}|^p dt$$

as $n, n_o \to \infty$, by the Monotone Convergence Theorem $\sum j^{-1/\alpha}\varepsilon_j\bar{f}(\tau_j^f)$ converges in $L^p_{AC^p[a,b]}$ if and only if

$$\lim_{n\to\infty} E||\sum_{j=1}^n j^{-1/\alpha}\varepsilon_j\bar{f}\,(\tau_j^f)||^p < \infty.$$

Case $\alpha < p$. By Lemma 6.2

$$\lim_{n\to\infty} E \,\Big\|\sum_{j=1}^{n} j^{-1/\alpha}\varepsilon_j \bar{f}\,(\tau_j^f)\Big\|^p \underset{C}{\sim} \int_a^b E\big|g(t,\tau_1^f)\,\|f(\tau_1^f)\|^{-1}\big|^p\,dt$$

$$= [m_f(S)]^{-1}\int_a^b\!\!\int_S |g(t,s)|^p\|f(s)\|^{\alpha-p}\,dm(s)\,dt = [m_f(S)]^{-1}\int_S \|f(s)\|^{\alpha}dm(s) = 1.$$

Therefore the condition $f \in L^{\alpha}_{AC^p[a,b]}$ is also sufficient for $f \in S^{\alpha}_{AC^p[a,b]}$.

Case $\alpha = p$. By Lemma 6.2

$$\lim_{n\to\infty} E\Big\|\sum_{j=1}^{n} j^{-1/\alpha}\varepsilon_j\bar{f}(\tau_j^f)\Big\|^p$$

$$\underset{C}{\sim} \quad [m_f(S)]^{-1}\int_a^b\!\!\int_S |g(t,s)|^{\alpha}\Big[1+\log_+\frac{|g(t,s)|^{\alpha}m_f(S)}{\|f(s)\|^{\alpha}\int|g(t,v)|^{\alpha}dm}\Big]\,dm(s)\,dt$$

which together with (6.8) ends this proof.

Case $\alpha > p$. By Lemma 6.2

$$\lim_{n\to\infty} E\,\Big\|\sum_{j=1}^{n} j^{-1/\alpha}\varepsilon_j\bar{f}(\tau_j^f)\Big\|^p$$

$$\underset{C}{\sim} [m_f(S)]^{-p/\alpha}\Big[\int_a^b \int_S |g(t,s)|^{\alpha}\,dm(s)\Big]^{p/\alpha}\,dt,$$

which in conjunction with (6.8) completes the proof of Theorem 6.1.

Remarks:

Theorem 6.1 with appropriate modifications gives conditions for paths to have (n-1) continuous derivatives with the (n-1)th derivative in $AC^p[a,b]$.

An alternative proof of Theeorm 6.1 can be obtained by an observation that $AC^p[a,b]$ is isomorphic with $\mathbb{R}\times L^p[a,b]$ $(x \to (x(a),\frac{dx}{dt}))$, and by the fact that a full characterization of stable measures on L^p-spaces is known (see [18], [3] and [14] for $p \neq \alpha$ and [27] for $p = \alpha$). The proof given here, which is a straightforward application of Theorem 4.2, uses the same argument for all cases of p and α and is self-contained.

The following result gives a full characterization of harmonizable SαS processes with absolutely continuous sample paths. Cambanis and Miller [3], using different methods, solved the case $\alpha > 1$.

Corollary 6.3. *Let M be an SαS random measure on the Borel σ-algebra of $\mathbb{R}$ with the finite control measure m. Then $X(t) = \int_{-\infty}^{\infty} e^{its} dM(s)$, $t \in [a,b]$, has a modification with sample paths in $AC^p[a,b]$, $1 \leq p < \infty$, if and only if*

$$(6.9) \qquad \int_{-\infty}^{\infty} |s|^{\alpha} \, dm(s) < \infty.$$

Proof. Since $\mathbb{R}^2$ and $\mathbb{C}$ are isomorphic Lemma 6.2 can be immediately extended to the case of complex valued random variables by considering two-dimensional random vectors instead of real random variables. Therefore Theorem 6.1 remains true when we replace a real valued function h by a complex one. In our case

$$h(t,s) = e^{its} \quad \text{and} \quad \frac{\partial h}{\partial t} = ise^{its}.$$

It is elementary to check that in all cases of p and α Theorem 6.1 yields the same condition (6.9).

Another important class of stable processes which is disjoint from the class of harmonizable ones (see Cambanis and Soltani [6]), is the class of SαS processes having the moving average representation.

Corollary 6.4. *Let* k: $\mathbb{R} \to \mathbb{R}$ *be an absolutely continuous function on every finite interval and such that* $\int_{-\infty}^{\infty} |k(s)|^{\alpha} ds < \infty$. *Let* $X(t) = \int_{-\infty}^{\infty} k(t-s) dM(s)$, $t \in \mathbb{R}$, *where* M *is an* SαS *random measure defined on Borel bounded subsets of* $\mathbb{R}$ *with the Lebesgue measure as the control measure.*

Then X *has a modification with sample paths in* $AC^p[a,b]$ *for every* $-\infty < a < b < \infty$ *if and only if*

$$\int_{-\infty}^{\infty} (k_p(u))^{\alpha} du < \infty \quad \text{if } \alpha < p$$

$$\int_0^1 \int_{-\infty}^{\infty} \left|\frac{dk}{ds}\right|^{\alpha} \left(1 + \log_+ \frac{\left|\frac{dk}{ds}\right|}{k_{\alpha}(s+t)}\right) ds dt < \infty \text{ if } \alpha = p$$

and

$$\int_{-\infty}^{\infty} \left|\frac{dk}{ds}\right|^{\alpha} ds < \infty \quad \text{if } \alpha > p,$$

where $k_p(u) = \left(\int_u^{u+1} \left|\frac{dk}{ds}\right|^p ds\right)^{1/p}, \quad u \in \mathbb{R}.$

Proof. Since X is a strictly stationary process it is enough to show that $\{X(t): t \in [0,1]\}$ has a modification with sample paths in $AC^p[a,b]$ if and only if the above conditions hold. Define $h_0(t,s) = k(t-s)$, $t,s \in \mathbb{R}$. Then $\frac{\partial h_0}{\partial t}(t,s) = \frac{dk}{ds}(t-s)$ and it is easy to check that the condition $Q_{\alpha,p}(\frac{\partial h_0}{\partial t}) < \infty$ is equivalent to the above conditions for k.

7. Bounds for moments of a double α-stable stochastic integral.

Let $h:[0,1] \times [0,1] \to \mathbb{R}$ be a jointly measurable function such that $h(t,s) = 0$ for $s \geq t$. Let M be an SαS random measure on the Borel σ-algebra of $[0,1]$.

McConnell and Taqqu [20] have proved that a double stochastic integral

$$(7.1) \qquad J(h) = \int_0^1 \int_0^1 h(t,s)dM(s)dM(t)$$

exists as the limit in L^p $(p < \alpha)$ of integrals of "dyadic" functions if and only if

$$(7.2) \qquad P\{\int_0^1 | \int_0^1 h(t,s)dM(s)|^\alpha \, dt < \infty\} = 1$$

and in this case

$$(7.3) \qquad (E|J(h)|^p)^{1/p} \underset{C}{\sim} \rho_{\alpha,p}(h),$$

where $\rho_{\alpha,p}(h) = \{E[\int_0^1 | \int_0^1 h(t,s)dM(s)|^\alpha dt]^{p/\alpha}\}^{1/p}$,

$C = C(\alpha,p)$ and $p < \alpha$. Moreover, $\rho_{\alpha,p}$ is a complete norm (quasi-norm if $p < 1$) on the space of all functions h such that $J(h)$ exists.

At the same time Rosinski and Woyczynski [27] studied double α-stable integrals as iterated Ito-type stochastic integrals and proved that the finiteness of

$$(7.4) \qquad N_\alpha^\alpha(h) = \int_0^1\int_0^1 |h(t,s)|^\alpha[1 + \log_+ \frac{|h(t,s)|^\alpha \int_0^1|h(u,v)|^\alpha \, dudv}{\int_0^1|h(t,v)|^\alpha dv \int_0^1|h(u,s)|^\alpha du}]dt\, ds$$

is necessary and sufficient for the existence of $J(h)$ in this sense. They have proved also the equivalence of (7.2) and $N_\alpha(h) < \infty$. This shows, in particular, that both approaches to define a double α-stable integral are equivalent.

A natural problem which arises here is the relation between the norm $\rho_{\alpha,p}$ and the functional N_α. We shall prove that $\rho_{\alpha,p} \underset{C}{\sim} N_\alpha$, where $C = C(\alpha,p)$. This in conjunction with (7.3) yields definitive bounds for moments of $J(h)$.

Let now $h: [0,1]^2 \to \mathbb{R}$ be a jointly measurable function such that for every $t \in [0,1]$ $h(t,\cdot) \in L^\alpha[0,1]$. By Proposition 6.1 in [27]

$$X(t) = \int_0^1 h(t,s)dM(s), \qquad t \in [0,1]$$

can be defined as a measurable stochastic process and by (7.2) $X(\cdot,\omega) \in L^\alpha[0,1]$ for almost all ω. Therefore, $\int_0^1 \int_0^1 |h(t,s)|^\alpha ds\, dt < \infty$ (cf. [28]).

The lemma below justifies the interchanging of stochastic and usual integration and for the case $\alpha > 1$ has been proved in [4] (Theorem 4.6) and in [20], Lemma 4.4. We give here a simpler and shorter proof of this result.

Lemma 7.1. *Let $\alpha \geq 1$ and let h and X be as above. Then for every* $\phi \in L^{\alpha'}[0,1]$, $(\frac{1}{\alpha} + \frac{1}{\alpha'} = 1)$

$$(7.5) \qquad \int_0^1 \phi(t)X(t)dt = \int_0^1 [\int_0^1 \phi(t)h(t,s)dt]dM(s) \quad \text{a.s.}$$

Proof. Let $\{U_j\}$ be a sequence of i.i.d. random variables uniformly distributed in $[0,1]$ and defined on an auxiliary probability space (Ω',P'), so that $\{U_j\}$ is independent of $\{X(t): t \in [0,1]\}$. For every fixed $\omega \in \Omega$ such that $X(\cdot,\omega) \in L^\alpha[0,1]$ random variables $\Omega' \ni \omega' \to \phi(U_j(\omega'))X(U_j(\omega'),\omega)$ are i.i.d. and $E' |\phi(U_j)X(U_j,\omega)| = \int_0^1 |\phi(t)X(t,\omega)|dt < \infty$. Therefore Kolmogorov's

SLLN yields the P'-a.s. convergence:

$$\frac{1}{n}\sum_{j=1}^{n}\phi(U_j)X(U_j,\omega) \to E'\phi(U_1)X(U_1,\omega) = \int_0^1 \phi(t)X(t,\omega)dt.$$

By Fubini's theorem, for almost all $\omega' \in \Omega'$

$$\frac{1}{n}\sum_{j=1}^{n}\phi(U_j(\omega'))X(U_j(\omega'),\cdot) \to \int_0^1 \phi(t)X(t)dt \tag{7.6}$$

P-a.s. on Ω.

Define now i.i.d. random elements $Y_j\colon \Omega' \to L^{\alpha}[0,1]$ by $[Y_j(\omega')](s) = \phi(U_j(\omega'))h(U_j(\omega'),s)$. Then

$$E||Y_j||_{L^{\alpha}[0,1]} = \int_0^1 ||\phi(t)h(t,\cdot)||_{L^{\alpha}[0,1]}\, dt$$

$$= \int_0^1 [\int_0^1 |\phi(t)|^{\alpha}\, |h(t,s)|^{\alpha} ds]^{1/\alpha}\, dt$$

$$= \int_0^1 |\phi(t)|\, [\int_0^1 |h(t,s)|^{\alpha}\, ds]^{1/\alpha}\, dt$$

$$\leq [\int_0^1 |\phi(t)|^{\alpha'} dt]^{1/\alpha'} [\int_0^1\int_0^1 |h(t,s)|^{\alpha}\, dsdt]^{1/\alpha} < \infty.$$

By SLLN in $L^{\alpha}[0,1]$

$$\frac{1}{n}\sum_{j=1}^{n} Y_j \to E'Y_1 = \int_0^1 \phi(t)h(t,\cdot)dt$$

P'-a.s. Therefore for almost all $\omega' \in \Omega'$

$$\frac{1}{n}\sum_{j=1}^{n}\phi(U_j(\omega'))X(U_j(\omega'),\cdot) = \int_0^1 [\frac{1}{n}\sum_{j=1}^{n}\phi(U_j(\omega'))h(U_j(\omega'),s)]dM(s) =$$

(7.7)

$$= \int_0^1 [\frac{1}{n}\sum_{j=1}^{n} Y_j(\omega')]dM \xrightarrow{P} \int_0^1 [\int_0^1 \phi(t)h(t,s)dt]dM(s).$$

Since there exists $\omega' \in \Omega'$ for which both (7.6) and (7.7) hold the proof is complete.

Corollary 7.2. *Let* $h: [0,1]^2 \to \mathbb{R}$ *and* X *be as in Lemma* 7.1. *Then the function* $f: [0,1] \to L^{\alpha}[0,1]$ *defined by* $[f(s)](t) = h(t,s)$ *belongs to* $S^{\alpha}_{L^{\alpha}[0,1]}$ *and*

$$X = \int_0^1 f(s)dM(s), \quad \text{a.s.}$$

Proof. By (7.5) for every $\phi \in (L^{\alpha}[0,1])'$ $\langle X,\phi\rangle = \int_0^1 \langle f,\phi\rangle dM$ a.s.

Thus Proposition 3.1 completes the proof.

Theorem 7.3. *Let* $1 \le \alpha < 2$ *and* $p < \alpha$. *Then there exists* $C = C(\alpha,p)$ *such that for every* h

$$(E|J(h)|^p)^{1/p} \underset{C}{\sim} N_\alpha(h).$$

Proof. In view of (7.3) it is sufficient to prove $\rho_{\alpha,p} \tilde{C} N_\alpha$. By Corollary 7.2 and Theorem 4.2

$$\rho_{\alpha,p}(h) = [E(\int_0^1 |X(t)|^\alpha dt)^{p/\alpha}]^{1/p} = (E\|X\|^p_{L^\alpha[0,1]})^{1/p} = (E\|\int_0^1 fdM\|^p_{L^\alpha[0,1]})^{1/p}$$

$$\underset{C}{\sim} (\int_0^1 \|f(s)\|^\alpha_{L^\alpha[0,1]} ds)^{1/\alpha}[1+(E\|\sum_{j=1}^{\infty} j^{-1/\alpha}\varepsilon_j \bar{f}(\xi_j^f)\|^\alpha_{L^\alpha[0,1]})^{1/\alpha}].$$

Here $\int_0^1 ||f(s)||^{\alpha}_{L^{\alpha}[0,1]} ds = \int_0^1\int_0^1 |h(t,s)|^{\alpha} dtds$ and by Lemma 6.2

$$E\,||\sum_{j=1}^{\infty} j^{-1/\alpha}\varepsilon_j \bar{f}(\tau_j^f)||^{\alpha}_{L^{\alpha}[0,1]} = E\int_0^1 |\sum_{j=1}^{\infty} j^{-1/\alpha}\varepsilon_j ||f(\tau_j^f)||^{-1}_{L^{\alpha}[0,1]} h(t,\tau_j^f)|^{\alpha} dt$$

$$\tilde{C}\int_0^1 E||f(\tau_1^f)||^{-\alpha}_{L^{\alpha}[0,1]} |h(t,\tau_1^f)| \; [1 + \log_+ \frac{||f(\tau_1^f)||^{-\alpha}_{L^{\alpha}[0,1]} |h(t,\tau_1^f)|^{\alpha}}{E||f(\tau_1^f)||^{-\alpha}_{L^{\alpha}[0,1]} |h(t,\tau_1^f)|^{\alpha}}] \, dt$$

$$= (\int_0^1 \int_0^1 |h(t,s)|^{\alpha} dt\, ds)^{-1} N_{\alpha}^{\alpha}(h)$$

which finishes the proof.

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